6-1 Operations on Functions

Find $(f+g)(x)$, $(f-g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Indicate any restrictions in domain or range.

1. $f(x) = x + 2$
$g(x) = 3x - 1$

SOLUTION:

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= (x+2) + (3x-1)\\ &= 4x + 1\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= (x+2) - (3x-1)\\ &= -2x + 3\end{aligned}$$

$$\begin{aligned}(f\cdot g)(x) &= f(x)\cdot g(x)\\ &= (x+2)\cdot(3x-1)\\ &= 3x^2 + 5x - 2\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, g(x) \neq 0\\ &= \frac{x+2}{3x-1},\ x \neq \frac{1}{3}\end{aligned}$$

2. $f(x) = x^2 - 5$
$g(x) = -x + 8$

SOLUTION:

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= (x^2 - 5) + (-x + 8)\\ &= x^2 - x + 3\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= (x^2 - 5) - (-x + 8)\\ &= x^2 + x - 13\end{aligned}$$

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x)\\ &= (x^2 - 5) \cdot (-x + 8)\\ &= -x^3 + 8x^2 + 5x - 40\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, g(x) \neq 0\\ &= \frac{x^2 - 5}{-x + 8}, x \neq 8\end{aligned}$$

6-1 Operations on Functions

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.

3. f = {(2, 5), (6, 10), (12, 9), (7, 6)}
g = {(9, 11), (6, 15), (10, 13), (5, 8)}

SOLUTION:

The range of $g(x)$ is not a subset of the domain of $f(x)$.

So, $f \circ g$ is undefined.

$$[g \circ f](x) = g[f(x)]$$

Therefore:

$$g[f(2)] = g(5) = 8$$
$$g[f(6)] = g(10) = 13$$
$$g[f(12)] = g(9) = 11$$
$$g[f(7)] = g(6) = 15$$

$$g \circ f = \{(2,8),(6,13),(12,11),(7,15)\}$$

4. f = {(–5, 4), (14, 8), (12, 1), (0, –3)}
g = {(–2, –4), (–3, 2), (–1, 4), (5, –6)}

SOLUTION:

The range of $g(x)$ is not a subset of the domain of $f(x)$.

So, $f \circ g$ is undefined.

$$[g \circ f](x) = g[f(x)]$$

Therefore:

$$g[f(0)] = g(-3) = 2$$
$$g \circ f = \{(0,2)\}$$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

5. $f(x) = -3x$
$g(x) = 5x - 6$

SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f[5x-6] \\ &= -3(5x-6) \\ &= -15x+18\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g[-3x] \\ &= 5(-3x)-6 \\ &= -15x-6\end{aligned}$$

6. $f(x) = x + 4$
$g(x) = x^2 + 3x - 10$

SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f[x^2+3x-10] \\ &= x^2+3x-10+4 \\ &= x^2+3x-6\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g[x+4] \\ &= (x+4)^2+3(x+4)-10 \\ &= x^2+11x+18\end{aligned}$$

7. **CCSS MODELING** Dora has 8% of her earnings deducted from her paycheck for a college savings plan. She can choose to take the deduction either before taxes are withheld, which reduces her taxable income, or after taxes are withheld. Dora's tax rate is 17.5%. If her pay before taxes and deductions is $950, will she save more money if the deductions are taken before or after taxes are withheld? Explain.

SOLUTION:

Either way, she will have $228.95 taken from her paycheck. If she takes the college savings plan deduction before taxes, $76 will go to her college plan and $152.95 will go to taxes. If she takes the college savings plan deduction after taxes, only $62.70 will go to her college plan and $166.25 will go to taxes.

Find $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Indicate any restrictions in domain or range.

8. $f(x)=2x$
$g(x)=-4x+5$

SOLUTION:

$$\begin{aligned}(f+g)(x)&=f(x)+g(x)\\&=(2x)+(-4x+5)\\&=-2x+5\end{aligned}$$

$$\begin{aligned}(f-g)(x)&=f(x)-g(x)\\&=2x-(-4x+5)\\&=6x-5\end{aligned}$$

$$\begin{aligned}(f\cdot g)(x)&=f(x)\cdot g(x)\\&=(2x)\cdot(-4x+5)\\&=-8x^2+10x\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(x)&=\frac{f(x)}{g(x)},g(x)\neq 0\\&=\frac{2x}{-4x+5},\ x\neq\frac{5}{4}\end{aligned}$$

9. $f(x) = x - 1$
$g(x) = 5x - 2$

SOLUTION:

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= (x-1) + (5x-2)\\ &= 6x - 3\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= (x-1) - (5x-2)\\ &= -4x + 1\end{aligned}$$

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x)\\ &= (x-1) \cdot (5x-2)\\ &= 5x^2 - 7x + 2\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, g(x) \neq 0\\ &= \frac{x-1}{5x-2}, x \neq \frac{2}{5}\end{aligned}$$

10. $f(x) = x^2$
$g(x) = -x + 1$

SOLUTION:

$$
\begin{aligned}
(f+g)(x) &= f(x) + g(x) \\
&= x^2 - x + 1 \\
(f-g)(x) &= f(x) - g(x) \\
&= x^2 + x - 1 \\
(f \cdot g)(x) &= f(x) \cdot g(x) \\
&= (x^2) \cdot (-x+1) \\
&= -x^3 + x^2 \\
\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, g(x) \neq 0 \\
&= \frac{x^2}{-x+1},\ x \neq 1
\end{aligned}
$$

11. $f(x) = 3x$
$g(x) = -2x + 6$

SOLUTION:

$$
\begin{aligned}
(f+g)(x) &= f(x) + g(x) \\
&= 3x - 2x + 6 \\
&= x + 6 \\
(f-g)(x) &= f(x) - g(x) \\
&= 3x + 2x - 6 \\
&= 5x - 6 \\
(f \cdot g)(x) &= f(x) \cdot g(x) \\
&= (3x) \cdot (-2x+6) \\
&= -6x^2 + 18x \\
\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, g(x) \neq 0 \\
&= \frac{3x}{-2x+6},\ x \neq 3
\end{aligned}
$$

12. $f(x) = x - 2$
$g(x) = 2x - 7$

SOLUTION:

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= x - 2 + 2x - 7\\ &= 3x - 9\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= x - 2 - (2x - 7)\\ &= -x + 5\end{aligned}$$

$$\begin{aligned}(f\cdot g)(x) &= f(x)\cdot g(x)\\ &= (x-2)\cdot(2x-7)\\ &= 2x^2 - 11x + 14\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)},\ g(x) \neq 0\\ &= \frac{x-2}{2x-7}, x \neq \frac{7}{2}\end{aligned}$$

13. $f(x) = x^2$
$g(x) = x - 5$

SOLUTION:

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= x^2 + x - 5\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= x^2 - x + 5\end{aligned}$$

$$\begin{aligned}(f\cdot g)(x) &= f(x)\cdot g(x)\\ &= x^3 - 5x^2\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, g(x) \neq 0\\ &= \frac{x^2}{x-5}, x \neq 5\end{aligned}$$

6-1 Operations on Functions

14. $f(x) = -x^2 + 6$
$g(x) = 2x^2 + 3x - 5$

SOLUTION:

$$\begin{aligned}(f+g)(x) &= f(x) + g(x)\\ &= x^2 + 3x + 1\end{aligned}$$

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= -x^2 + 6 - (2x^2 + 3x - 5)\\ &= -3x^2 - 3x + 11\end{aligned}$$

$$\begin{aligned}(f\cdot g)(x) &= f(x)\cdot g(x)\\ &= (-x^2 + 6)\cdot(2x^2 + 3x - 5)\\ &= -2x^4 - 3x^3 + 17x^2 + 18x - 30\end{aligned}$$

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, g(x) \neq 0\\ &= \frac{-x^2 + 6}{2x^2 + 3x - 5},\ 2x^2 + 3x - 5 \neq 0\\ &= \frac{-x^2 + 6}{2x^2 + 3x - 5},\ x \neq 1 \text{ or } -\frac{5}{2}\end{aligned}$$

15. $f(x) = 3x^2 - 4$
$g(x) = x^2 - 8x + 4$

SOLUTION:

$$(f+g)(x) = f(x) + g(x)$$
$$= 4x^2 - 8x$$
$$(f-g)(x) = f(x) - g(x)$$
$$= 2x^2 + 8x - 8$$
$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$= (3x^2 - 4) \cdot (x^2 - 8x + 4)$$
$$= 3x^4 - 24x^3 + 8x^2 + 32x - 16$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$$
$$= \frac{3x^2 - 4}{x^2 - 8x + 4}, x^2 - 8x + 4 \neq 0$$
$$= \frac{3x^2 - 4}{x^2 - 8x + 4}, x \neq 4 \pm 2\sqrt{3}$$

16. **POPULATION** In a particular county, the population of the two largest cities can be modeled by $f(x) = 200x + 25$ and $g(x) = 175x - 15$, where x is the number of years since 2000 and the population is in thousands.

a. What is the population of the two cities combined after any number of years?

b. What is the difference in the populations of the two cities?

SOLUTION:

a. The population of the cities after x years is the sum of the individual populations.

$$(f+g)(x) = f(x) + g(x)$$
$$= 200x + 25 + 175x - 15$$
$$= 375x + 10$$

b. The difference in the populations of the cities is given by:

$$(f-g)(x) = 200x + 25 - (175x - 15)$$
$$= 25x + 40$$

For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.

17. $f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}$
$g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}$

SOLUTION:

$[f \circ g](x) = f[g(x)]$

Therefore:

$$\begin{aligned}[f \circ g](-4) &= f[g(-4)] \\ &= f(0) \\ &= 4\end{aligned}$$

$$f \circ g = \{(-4, 4)\}$$

$[g \circ f](x) = g[f(x)]$

Therefore:

$$\begin{aligned}[g \circ f](-8) &= g[f(-8)] \\ &= g(-4) \\ &= 0\end{aligned}$$

$$\begin{aligned}[g \circ f](0) &= g[f(0)] \\ &= g(4) \\ &= -4\end{aligned}$$

$$\begin{aligned}[g \circ f](2) &= g[f(2)] \\ &= g(6) \\ &= -5\end{aligned}$$

$$\begin{aligned}[g \circ f](-6) &= g[f(-6)] \\ &= g(-2) \\ &= -1\end{aligned}$$

$$g \circ f = \{(-8, 0), (0, -4), (2, -5), (-6, -1)\}$$

18. f = {(–7, 0), (4, 5), (8, 12), (–3, 6)}
g = {(6, 8), (–12, –5), (0, 5), (5, 1)}

SOLUTION:

$$[f \circ g](x) = f[g(x)]$$
$$\begin{aligned}[f \circ g](6) &= f[g(6)] \\ &= f(8) \\ &= 12\end{aligned}$$
$$f \circ g = \{(6,12)\}$$
$$[g \circ f](x) = g[f(x)]$$
$$\begin{aligned}[g \circ f](-7) &= g[f(-7)] \\ &= g(0) \\ &= 5\end{aligned}$$
$$\begin{aligned}[g \circ f](4) &= g[f(4)] \\ &= g(5) \\ &= 1\end{aligned}$$
$$\begin{aligned}[g \circ f](-3) &= g[f(-3)] \\ &= g(6) \\ &= 8\end{aligned}$$
$$g \circ f = \{(-7,5),(4,1),(-3,8)\}$$

19. f = {(5, 13), (–4, –2), (–8, –11), (3, 1)}
g = {(–8, 2), (–4, 1), (3, –3), (5, 7)}

SOLUTION:

The range of $g(x)$ is not a subset of the domain of $f(x)$.
So, $f \circ g$ is undefined.
The range of $f(x)$ is not a subset of the domain of $g(x)$.
So, $g \circ f$ is undefined.

20. $f = \{(-4, -14), (0, -6), (-6, -18), (2, -2)\}$
$g = \{(-6, 1), (-18, 13), (-14, 9), (-2, -3)\}$

SOLUTION:

The range of $g(x)$ is not a subset of the domain of $f(x)$.

So, $f \circ g$ is undefined.

$$[g \circ f](x) = g[f(x)]$$

$$\begin{aligned}[g \circ f](-4) &= g[f(-4)] \\ &= g(-14) \\ &= 9\end{aligned}$$

$$\begin{aligned}[g \circ f](0) &= g[f(0)] \\ &= g(-6) \\ &= 1\end{aligned}$$

$$\begin{aligned}[g \circ f](-6) &= g[f(-6)] \\ &= g(-18) \\ &= 13\end{aligned}$$

$$\begin{aligned}[g \circ f](2) &= g[f(2)] \\ &= g(-2) \\ &= -3\end{aligned}$$

$$g \circ f = \{(-4,9),(0,1),(-6,13),(2,-3)\}$$

21. f = {(–15, –5), (–4, 12), (1, 7), (3, 9)}
g = {(3, –9), (7, 2), (8, –6), (12, 0)}

SOLUTION:

The range of $g(x)$ is not a subset of the domain of $f(x)$.

So, $f \circ g$ is undefined.

$$[g \circ f](x) = g[f(x)]$$
$$\begin{aligned}[g \circ f](-4) &= g[f(-4)] \\ &= g(12) \\ &= 0\end{aligned}$$
$$\begin{aligned}[g \circ f](1) &= g[f(1)] \\ &= g(7) \\ &= 2\end{aligned}$$
$$g \circ f = \{(-4, 0), (1, 2)\}$$

22. f = {(–1, 11), (2, –2), (5, –7), (4, –4)}
g = {(5, –4), (4, –3), (–1, 2), (2, 3)}

SOLUTION:

$$[f \circ g](x) = f[g(x)]$$
$$\begin{aligned}[f \circ g](-1) &= f[g(-1)] \\ &= f(2) \\ &= -2\end{aligned}$$
$$f \circ g = \{(-1, -2)\}$$

The range of $f(x)$ is not a subset of the domain of $g(x)$.

So, $g \circ f$ is undefined.

23. f = {(7, –3), (–10, –3), (–7, -8), (–3, 6)}
g = {(4, –3), (3, –7), (9, 8), (–4, –4)}

SOLUTION:

$$[f \circ g](x) = f[g(x)]$$
$$[f \circ g](4) = f[g(4)]$$
$$= f(-3)$$
$$= 6$$
$$[f \circ g](3) = f[g(3)]$$
$$= f(-7)$$
$$= -8$$
$$f \circ g = \{(4, 6), (3, -8)\}$$

The range of $f(x)$ is not a subset of the domain of $g(x)$.
So, $g \circ f$ is undefined.

24. f = {(1, –1), (2, –2), (3, –3), (4, –4)}
g = {(1, –4), (2, –3), (3, –2), (4, –1)}

SOLUTION:

The range of $g(x)$ is not a subset of the domain of $f(x)$.
So, $f \circ g$ is undefined.
The range of $f(x)$ is not a subset of the domain of $g(x)$.
So, $g \circ f$ is undefined.

25. $f = \{(-4, -1), (-2, 6), (-1, 10), (4, 11)\}$
$g = \{(-1, 5), (3, -4), (6, 4), (10, 8)\}$

SOLUTION:

$$[f \circ g](x) = f[g(x)]$$
$$[f \circ g](3) = f[g(3)]$$
$$= f(-4)$$
$$= -1$$
$$[f \circ g](6) = f[g(6)]$$
$$= f(4)$$
$$= 11$$
$$f \circ g = \{(3,-1),(6,11)\}$$
$$[g \circ f](x) = g[f(x)]$$
$$[g \circ f](-4) = g[f(-4)]$$
$$= g(-1)$$
$$= 5$$
$$[g \circ f](-2) = g[f(-2)]$$
$$= g(6)$$
$$= 4$$
$$[g \circ f](-1) = g[f(-1)]$$
$$= g(10)$$
$$= 8$$
$$g \circ f = \{(-4,5),(-2,4),(-1,8)\}$$

26. $f = \{(12, -3), (9, -2), (8, -1), (6, 3)\}$
$g = \{(-1, 5), (-2, 6), (-3, -1), (-4, 8)\}$

SOLUTION:

$$[f \circ g](x) = f[g(x)]$$
$$\begin{aligned}[f \circ g](-2) &= f[g(-2)] \\ &= f(6) \\ &= 3\end{aligned}$$
$$\begin{aligned}[f \circ g](-4) &= f[g(-4)] \\ &= f(8) \\ &= -1\end{aligned}$$
$$f \circ g = \{(-2,3),(-4,-1)\}$$
$$[g \circ f](x) = g[f(x)]$$
$$\begin{aligned}[g \circ f](12) &= g[f(12)] \\ &= g(-3) \\ &= -1\end{aligned}$$
$$\begin{aligned}[g \circ f](9) &= g[f(9)] \\ &= g(-2) \\ &= 6\end{aligned}$$
$$\begin{aligned}[g \circ f](8) &= g[f(8)] \\ &= g(-1) \\ &= 5\end{aligned}$$
$$g \circ f = \{(12,-1),(9,6),(8,5)\}$$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

27. $f(x) = 2x$
$g(x) = x + 5$

SOLUTION:

$$\begin{aligned} f \circ g(x) &= f[g(x)] \\ &= f(x+5) \\ &= 2(x+5) \\ &= 2x + 10 \end{aligned}$$

$$\begin{aligned} [g \circ f](x) &= g[f(x)] \\ &= g(2x) \\ &= 2x + 5 \end{aligned}$$

For $[f \circ g](x)$, D = {all real numbers}, R = {all even numbers}
For $[g \circ f](x)$, D = {all real numbers}, R = {all odd numbers}

28. $f(x) = -3x$
$g(x) = -x + 8$

SOLUTION:

$$\begin{aligned} [f \circ g](x) &= f[g(x)] \\ &= f(-x+8) \\ &= -3(-x+8) \\ &= 3x - 24 \end{aligned}$$

$$\begin{aligned} [g \circ f](x) &= g[f(x)] \\ &= g(-3x) \\ &= -(-3x) + 8 \\ &= 3x + 8 \end{aligned}$$

For $[f \circ g](x)$, D = {all real numbers}, R = {all real numbers}
For $[g \circ f](x)$, D = {all real numbers}, R = {all real numbers}

29. $f(x) = x + 5$
$g(x) = 3x - 7$

SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(3x-7) \\ &= (3x-7)+5 \\ &= 3x-2\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(x+5) \\ &= 3(x+5)-7 \\ &= 3x+15-7 \\ &= 3x+8\end{aligned}$$

$For\ [f \circ g](x)$, D = {all real numbers}, R = {all real numbers}
$For\ [g \circ f](x)$, D = {all real numbers}, R = {all real numbers}

30. $f(x) = x - 4$
$g(x) = x^2 - 10$

SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(x^2-10) \\ &= (x^2-10)-4 \\ &= x^2-14\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(x-4) \\ &= (x-4)^2-10 \\ &= x^2-8x+6\end{aligned}$$

$For\ [f \circ g](x)$, D = {all real numbers}, R = {y | y ≥ –14}
$For\ [g \circ f](x)$, D = {all real numbers}, R = {y | y ≥ –10}

31. $f(x) = x^2 + 6x - 2$
$g(x) = x - 6$

SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(x-6) \\ &= (x-6)^2 + 6(x-6) - 2 \\ &= x^2 - 6x - 2\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(x^2 + 6x - 2) \\ &= x^2 + 6x - 2 - 6 \\ &= x^2 + 6x - 8\end{aligned}$$

For $[f \circ g](x)$, D = {all real numbers}, R = {y | y ≥ –11}
For $[g \circ f](x)$, D = {all real numbers}, R = {y | y ≥ –17}

32. $f(x) = 2x^2 - x + 1$
$g(x) = 4x + 3$

SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(4x+3) \\ &= 2(4x+3)^2 - (4x+3) + 1 \\ &= 32x^2 + 44x + 16\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(2x^2 - x + 1) \\ &= 4(2x^2 - x + 1) + 3 \\ &= 8x^2 - 4x + 7\end{aligned}$$

For $[f \circ g](x)$, D = {all real numbers}, R = {y | y ≥ 0.875}
For $[g \circ f](x)$, D = {all real numbers}, R = {y | y ≥ 6.5}

6-1 Operations on Functions

33. $f(x) = 4x - 1$
$g(x) = x^3 + 2$

SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(x^3 + 2) \\ &= 4(x^3 + 2) - 1 \\ &= 4x^3 + 7\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(4x - 1) \\ &= (4x - 1)^3 + 2 \\ &= 64x^3 - 48x^2 + 12x + 1\end{aligned}$$

For $[f \circ g](x)$, D = {all real numbers}, R = {all real numbers}
For $[g \circ f](x)$, D = {all real numbers}, R = {all real numbers}

34. $f(x) = x^2 + 3x + 1$
$g(x) = x^2$

SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(x^2) \\ &= (x^2)^2 + 3(x^2) + 1 \\ &= x^4 + 3x^2 + 1\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g[f(x)] \\ &= g(x^2 + 3x + 1) \\ &= (x^2 + 3x + 1)^2 \\ &= x^4 + 6x^3 + 11x^2 + 6x + 1\end{aligned}$$

For $[f \circ g](x)$, D = {all real numbers}, R = $\{y \mid y \geq 1\}$
For $[g \circ f](x)$, D = {all real numbers}, R = $\{y \mid y \geq 0\}$

35. $f(x) = 2x^2$
$g(x) = 8x^2 + 3x$

SOLUTION:

$$\begin{aligned}[f \circ g](x) &= f[g(x)] \\ &= f(8x^2 + 3x) \\ &= 2(8x^2 + 3x)^2 \\ &= 128x^4 + 96x^3 + 18x^2\end{aligned}$$

$$\begin{aligned}[g \circ f](x) &= g(f(x)) \\ &= g(2x^2) \\ &= 8(2x^2)^2 + 3(2x^2) \\ &= 32x^4 + 6x^2\end{aligned}$$

For $[f \circ g](x)$, D = {all real numbers}, R = {y | y ≥ 0}
For $[g \circ f](x)$, D = {all real numbers}, R = {y | y ≥ 0}

36. **FINANCE** A ceramics store manufactures and sells coffee mugs. The revenue $r(x)$ from the sale of x coffee mugs is given by $r(x) = 6.5x$. Suppose the function for the cost of manufacturing x coffee mugs is $c(x) = 0.75x + 1850$.

a. Write the profit function.

b. Find the profit on 500, 1000, and 5000 coffee mugs.

SOLUTION:

a. The profit function $P(x)$ is given by $P(x) = r(x) - c(x)$ where $r(x)$ is the revenue function and $c(x)$ is the cost function.
So:

$$\begin{aligned}P(x) &= 6.5x - (0.75x + 1850) \\ &= 5.75x - 1850\end{aligned}$$

b.

$$\begin{aligned}P(500) &= 5.75(500) - 1850 \\ &= \$1025\end{aligned}$$

$$\begin{aligned}P(1000) &= 5.75(1000) - 1850 \\ &= 5750 - 1850 \\ &= \$3900\end{aligned}$$

$$\begin{aligned}P(5000) &= 5.75(5000) - 1850 \\ &= \$26,900\end{aligned}$$

37. **CCSS SENSE-MAKING** Ms. Smith wants to buy an HDTV, which is on sale for 35% off the original price of $2299. The sales tax is 6.25%.

a. Write two functions representing the price after the discount $p(x)$ and the price after sales tax $t(x)$.

b. Which composition of functions represents the price of the HDTV, $[p \circ t](x)$ or $[t \circ p](x)$? Explain your reasoning.

c. How much will Ms. Smith pay for the HDTV?

SOLUTION:

a. Let $p(x)$ be the price after the discount where x is the original price.
Discount = 35%(x) = 0.35x
Therefore:

$$\begin{aligned} p(x) &= x - 0.35x \\ &= 0.65x \end{aligned}$$

Let $t(x)$ be the price after the sales tax.
Sales tax = 6.25%(x) = 0.0625x
Therefore:

$$\begin{aligned} t(x) &= x + 0.0625x \\ &= 1.0625x \end{aligned}$$

b. Since $[p \circ t](x) = [t \circ p](x)$, either function represents the price.

c.

$$\begin{aligned} [t \circ p](x) &= t[p(x)] \\ &= t(0.65x) \\ &= 1.0625(0.65x) \\ &= 0.690625x \end{aligned}$$

Substitute x = 2299.

$$\begin{aligned} [t \circ p](2299) &= 0.690625(2299) \\ &= 1587.75 \end{aligned}$$

Therefore, Mr. Smith will pay $1587.75 for HDTV.

6-1 Operations on Functions

Perform each operation if $f(x) = x^2 + x - 12$ and $g(x) = x - 3$. State the domain of the resulting function.

38. $(f - g)(x)$

SOLUTION:

$$\begin{aligned}(f-g)(x) &= f(x) - g(x)\\ &= x^2 + x - 12 - (x-3)\\ &= x^2 - 9\end{aligned}$$

D = {all real numbers}

39. $2(g \cdot f)(x)$

SOLUTION:

$$\begin{aligned}2(g \cdot f)(x) &= 2 \cdot g(x) \cdot f(x)\\ &= 2(x-3)(x^2 + x - 12)\\ &= 2x^3 - 4x^2 - 30x + 72\end{aligned}$$

D = {all real numbers}

40. $\left(\frac{f}{g}\right)(x)$

SOLUTION:

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}\\ &= \frac{x^2 + x - 12}{x-3},\ x \neq 3\\ &= \frac{(x+4)(x-3)}{(x-3)},\ x \neq 3\\ &= x + 4, \quad x \neq 3\end{aligned}$$

D = $\{x \mid x \neq 3\}$

If $f(x) = 5x$, $g(x) = -2x + 1$, and $h(x) = x^2 + 6x + 8$, find each value.

41. $f\left[g(-2)\right]$

SOLUTION:

$$\begin{aligned} f\left[g(x)\right] &= f[-2x+1] \\ &= 5(-2x+1) \\ &= -10x+5 \end{aligned}$$

Substitute $x = -2$.

$$\begin{aligned} f\left[g(-2)\right] &= -10(-2)+5 \\ &= 25 \end{aligned}$$

42. $g\left[h(3)\right]$

SOLUTION:

$$\begin{aligned} g\left[h(x)\right] &= g\left(x^2+6x+8\right) \\ &= -2\left(x^2+6x+8\right)+1 \\ &= -2x^2-12x-15 \end{aligned}$$

Substitute $x = 3$.

$$\begin{aligned} g\left[h(3)\right] &= -2(3)^2-12(3)-15 \\ &= -69 \end{aligned}$$

43. $h\left[f(-5)\right]$

SOLUTION:

$$\begin{aligned} h[f(x)] &= h[5x] \\ &= (5x)^2+6(5x)+8 \\ &= 25x^2+30x+8 \end{aligned}$$

Substitute $x = -5$.

$$\begin{aligned} h\left[f(-5)\right] &= 25(-5)^2+30(-5)+8 \\ &= 25(25)-150+8 \\ &= 625-150+8 \\ &= 483 \end{aligned}$$

44. $h\left[g(2)\right]$

SOLUTION:

$$\begin{aligned} h[g(x)] &= h[-2x+1] \\ &= (-2x+1)^2 + 6(-2x+1) + 8 \\ &= 4x^2 + 1 - 4x - 12x + 6 + 8 \\ &= 4x^2 - 16x + 15 \end{aligned}$$

Substitute $x = 2$.

$$\begin{aligned} h[g(2)] &= 4(2)^2 - 16(2) + 15 \\ &= 4(4) - 16(2) + 15 \\ &= 16 - 32 + 15 \\ &= 31 - 32 \\ &= -1 \end{aligned}$$

45. $f\left[h(-3)\right]$

SOLUTION:

$$\begin{aligned} f[h(x)] &= f\left[x^2 + 6x + 8\right] \\ &= 5\left(x^2 + 6x + 8\right) \\ &= 5x^2 + 30x + 40 \end{aligned}$$

Substitute $x = -3$.

$$\begin{aligned} f\left[h(-3)\right] &= 5(-3)^2 + 30(-3) + 40 \\ &= 45 - 90 + 40 \\ &= -5 \end{aligned}$$

46. $h[f(9)]$

SOLUTION:

$$\begin{aligned} h[f(x)] &= h(5x) \\ &= (5x)^2 + 6(5x) + 8 \\ &= 25x^2 + 30x + 8 \end{aligned}$$

Substitute $x = 9$.

$$\begin{aligned} h[f(9)] &= 25(9)^2 + 30(9) + 8 \\ &= 25(81) + 270 + 8 \\ &= 2025 + 270 + 8 \\ &= 2303 \end{aligned}$$

47. $f[g(3a)]$

SOLUTION:

$$\begin{aligned} f[g(3a)] &= f[-6a + 1] \\ &= 5(-6a + 1) \\ &= -30a + 5 \end{aligned}$$

48. $f[h(a+4)]$

SOLUTION:

$$\begin{aligned} f[h(a+4)] &= f\left[(a+4)^2 + 6(a+4) + 8\right] \\ &= f\left(a^2 + 14a + 48\right) \\ &= 5\left(a^2 + 14a + 48\right) \\ &= 5a^2 + 70a + 240 \end{aligned}$$

49. $g\left[f\left(a^2-a\right)\right]$

SOLUTION:

$$\begin{aligned} g\left[f\left(a^2-a\right)\right] &= g\left[5\left(a^2-a\right)\right] \\ &= g\left(5a^2-5a\right) \\ &= -2\left(5a^2-5a\right)+1 \\ &= -10a^2+10a+1 \end{aligned}$$

50. **MULTIPLE REPRESENTATIONS** Let $f(x) = x^2$ and $g(x) = x$.

a. TABULAR Make a table showing values for $f(x)$, $g(x)$, $(f + g)(x)$, and $(f - g)(x)$.

b. GRAPHICAL Graph $f(x)$, $g(x)$, and $(f + g)(x)$ on the same coordinate grid.

c. GRAPHICAL Graph $f(x)$, $g(x)$, and $(f - g)(x)$ on the same coordinate grid.

d. VERBAL Describe the relationship among the graphs of $f(x)$, $g(x)$, $(f + g)(x)$, and $(f - g)(x)$.

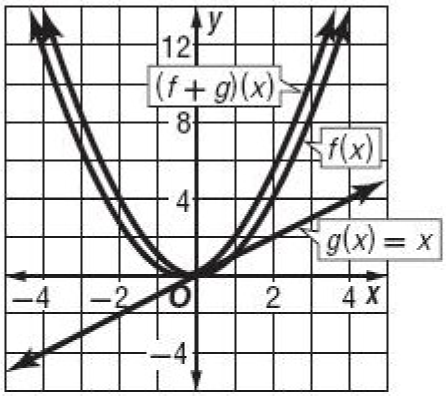
SOLUTION:

a.

x	$f(x) = x^2$	$g(x) = x$	$(f+g)(x) = x^2 + x$	$(f-g)(x) = x^2 - x$
–3	9	–3	6	12
–2	4	–2	2	6
–1	1	–1	0	2
0	0	0	0	0
1	1	1	2	0
2	4	2	6	2
3	9	3	12	6

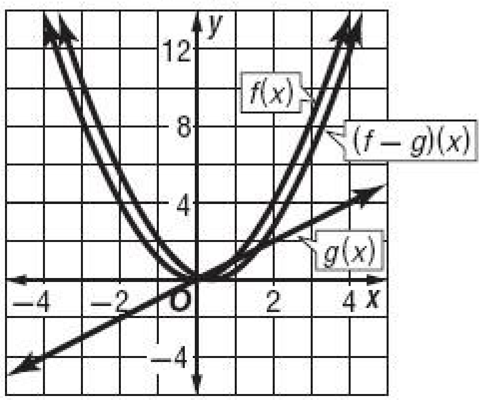
b.

$$\begin{aligned} (f+g)(x) &= f(x)+g(x) \\ &= x^2+x \end{aligned}$$



c.

$$(f-g)(x) = f(x) - g(x)$$
$$= x^2 - x$$



d. Sample answer: For each value of x, the vertical distance between the graph of $g(x)$ and the x-axis is the same as the vertical distance between the graphs of $f(x)$ and $(f+g)(x)$ and between $f(x)$ and $(f-g)(x)$.

51. **EMPLOYMENT** The number of women and men age 16 and over employed each year in the United States can be modeled by the following equations, where x is the number of years since 1994 and y is the number of people in thousands.
women: $y = 1086.4x + 56,610$
men: $y = 999.2x + 66,450$

a. Write a function that models the total number of men and women employed in the United States during this time.

b. If f is the function for the number of men, and g is the function for the number of women, what does $(f - g)(x)$ represent?

SOLUTION:

a. Add the functions.
The total number of men and women employed is given by

$$\begin{aligned} y(x) &= 1086.4x + 56,610 + 999.2x + 66,450 \\ &= 2085.6 + 123,060 \end{aligned}$$

b. The function $(f - g)(x)$ represents the difference in the number of men and women employed in the U.S.

If $f(x) = x + 2$, $g(x) = -4x + 3$, and $h(x) = x^2 - 2x + 1$, find each value.

52. $(f \cdot g \cdot h)(3)$

SOLUTION:

$$(f \cdot g \cdot h)(x) = f(x) \cdot g(x) \cdot h(x)$$

Substitute $x = 3$.

$$\begin{aligned} (f \cdot g \cdot h)(3) &= f(3) \cdot g(3) \cdot h(3) \\ &= (5)(-9)(4) \\ &= -180 \end{aligned}$$

6-1 Operations on Functions

53. $[(f+g)\cdot h](1)$

SOLUTION:

$$\begin{aligned}[(f+g)\cdot h](x) &= [f+g](x)\cdot h(x)\\ &= [f(x)+g(x)]\cdot h(x)\\ &= f(x)\cdot h(x)+g(x)\cdot h(x)\end{aligned}$$

Substitute $x = 1$.

$$\begin{aligned}[(f+g)\cdot h](1) &= f(1)\cdot h(1)+g(1)\cdot h(1)\\ &= (3)(0)+(-1)(0)\\ &= 0\end{aligned}$$

54. $\left(\frac{h}{fg}\right)(-6)$

SOLUTION:

$$\begin{aligned}\left(\frac{h}{fg}\right)(-6) &= \frac{h(-6)}{f(-6)\cdot g(-6)}\\ &= \frac{49}{(-4)(27)}\\ &= -\frac{49}{108}\end{aligned}$$

55. $[f\circ(g\circ h)](2)$

SOLUTION:

$$\begin{aligned}[f\circ(g\circ h)](2) &= [f\circ g](h(2))\\ &= [f\circ g](1)\\ &= f[g(1)]\\ &= f(-1)\\ &= 1\end{aligned}$$

6-1 Operations on Functions

56. $[g \circ (h \circ f)](-4)$

SOLUTION:

$$\begin{aligned}[g \circ (h \circ f)](-4) &= g\left(h\left[f(-4)\right]\right)\\ &= g\left(h(-2)\right)\\ &= g(9)\\ &= -36+3\\ &= -33\end{aligned}$$

57. $[h \circ (f \circ g)](5)$

SOLUTION:

$$\begin{aligned}[h \circ (f \circ g)](5) &= h\left[f\left(g(5)\right)\right]\\ &= h\left[f(-17)\right]\\ &= h(-15)\\ &= 256\end{aligned}$$

58. **MULTIPLE REPRESENTATIONS** You will explore $(f \cdot g)(x)$, $\left(\frac{f}{g}\right)(x)$, $[f \circ g](x)$, and $[g \circ f](x)$ if $f(x) = x^2 + 1$ and $g(x) = x - 3$.

a. Tabular Make a table showing values for $(f \cdot g)(x)$, $\left(\frac{f}{g}\right)(x)$, $[f \circ g](x)$, and $[g \circ f](x)$.

b. Graphical Use a graphing calculator to graph $(f \cdot g)(x)$, $\left(\frac{f}{g}\right)(x)$, $[f \circ g](x)$, and $[g \circ f](x)$ on the same coordinate plane.

c. Verbal Explain the relationship between $(f \cdot g)(x)$ and $\left(\frac{f}{g}\right)(x)$.

d. Graphical Use a graphing calculator to graph $[f \circ g](x)$, and $[g \circ f](x)$ on the same coordinate plane.

e. Verbal Explain the relationship between $[f \circ g](x)$, and $[g \circ f](x)$.

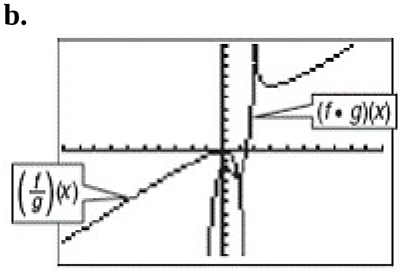
SOLUTION:

a.

$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$= \left(x^2+1\right) \cdot (x-3)$$
$$= x^3 - 3x^2 + x - 3$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$$
$$= \frac{x^2+1}{x-3}, x \neq 3$$
$$[f \circ g](x) = f[g(x)]$$
$$= f(x-3)$$
$$= (x-3)^2 + 1$$
$$= x^2 - 6x + 10$$
$$[g \circ f](x) = g[f(x)]$$
$$= g\left(x^2+1\right)$$
$$= \left(x^2+1\right) - 3$$
$$= x^2 - 2$$

x	$(f \cdot g)(x)$	$\left(\frac{f}{g}\right)(x)$	$[f \circ g](x)$	$[g \circ f](x)$
−3	−60	$-\frac{5}{3}$	37	7
−2	−25	−1	26	2
−1	−8	$-\frac{1}{2}$	17	−1
0	−3	$-\frac{1}{3}$	10	−2
1	−4	−1	5	−1
2	−5	−5	2	2
3	0	undef.	1	7

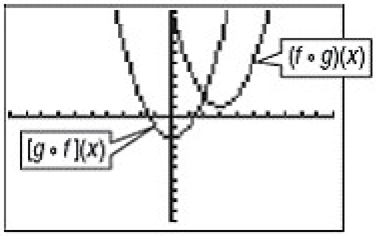
b.



[−20, 20] scl: 2 by [−20, 20] scl: 2

c. Sample answer: When x is 2 or 4, the functions are equal.

d.



[−10, 10] scl: 1 by [−10, 10] scl: 1

e. Sample answer: The functions are translations of the graph of $y = x^2$.

59. **OPEN ENDED** Write two functions $f(x)$ and $g(x)$ such that $(f \circ g)(4) = 0$.

SOLUTION:

Sample answer: Another way to write $(f \circ g)(x)$ is $f(g(x))$. Write a function for $g(x)$ first and then evaluate it. Let $g(x) = x + 5$. Then
$g(4) = 4 + 5$ or 9. Next, write a function for $f(x)$ such that when $x = 9$, $f(x) = 0$. Let $f(x) = x - 9$ then $f(g(4)) = 9 - 9$ or 0.

60. **CCSS CRITIQUE** Chris and Tobias are finding the composition $(f \circ g)(x)$, where $f(x) = x^2 + 2x - 8$ and $g(x) = x^2 + 8$. Is either of them correct? Explain your reasoning.

Chris

$$(f \circ g)(x) = f[g(x)]$$
$$= (x^2 + 8)^2 + 2x - 8$$
$$= x^4 + 16x^2 + 64 + 2x - 8$$
$$= x^4 + 16x^2 + 2x + 58$$

Tobias

$$(f \circ g)(x) = f[g(x)]$$
$$= (x^2 + 8)^2 + 2(x^2 + 8) - 8$$
$$= x^4 + 16x^2 + 64 + 2x^2 + 16 - 8$$
$$= x^4 + 18x^2 + 72$$

SOLUTION:

Tobias is correct. Chris did not substitute $g(x)$ for every x in $f(x)$.

61. **CHALLENGE** Given $f(x) = \sqrt{x^3}$ and $g(x) = \sqrt{x^6}$, determine the domain for each of the following.

a. $g(x) \cdot g(x)$

b. $f(x) \cdot f(x)$

SOLUTION:

a.

$$g(x) \cdot g(x) = \sqrt{x^6} \cdot \sqrt{x^6}$$
$$= x^6$$

$D = \{\text{all real numbers}\}$

b.

$$f(x) \cdot f(x) = \sqrt{x^3} \cdot \sqrt{x^3}$$
$$= x^3$$

Since $f(x)$ is defined for $x \geq 0$, the domain of $f(x) \cdot f(x)$ is $\{x | x \geq 0\}$.

62. **REASONING** State whether each statement is sometimes, always, or never true. Explain your reasoning.

a. The domain of two functions $f(x)$ and $g(x)$ that are composed $g[f(x)]$ is restricted by the domain of $f(x)$.

b. The domain of two functions $f(x)$ and $g(x)$ that are composed $g[f(x)]$ is restricted by the domain of $g(x)$.

SOLUTION:

a. Always; since the range is dependent on the domain, the domain of $g[f(x)]$ is restricted by the domain of $f(x)$.

b. Sometimes; when $f(x) = 4x$ and $g(x) = \sqrt{x}$, $g[f(x)] = \sqrt{4x}, x \geq 0$. The domain of $g(x)$ restricts the domain of $g[f(x)]$. When $f(x) = 4x^2$ and $g(x) = \sqrt{x}$, $g[f(x)] = \sqrt{4x^2}$. In this case, the domain of $g(x)$ does not restrict the domain of $g[f(x)]$.

63. **WRITING IN MATH** In the real world, why would you ever perform a composition of functions?

SOLUTION:

Sample answer: Many situations in the real world involve complex calculations in which multiple functions are used. In order to solve some problems, a composition of those functions may need to be used. For example, the product of a manufacturing plant may have to go through several processes in a particular order, in which each process is described by a function. By finding the composition, only one calculation must be made to find the solution to the problem.

64. What is the value of x in the equation $7(x - 4) = 44 - 11x$?

A 1

B 2

C 3

D 4

SOLUTION:

$$\begin{aligned} 7(x-4) &= 44-11x \\ 7x-28 &= 44-11x \\ 18x &= 72 \\ x &= 4 \end{aligned}$$

The correct choice is **D**.

65. If $g(x) = x^2 + 9x + 21$ and $h(x) = 2(x + 5)^2$, which is an equivalent form of $h(x) - g(x)$?

F $k(x) = -x^2 - 11x - 29$

G $k(x) = x^2 + 11x + 29$

H $k(x) = x + 4$

J $k(x) = x^2 + 7x + 11$

SOLUTION:

$$\begin{aligned} h(x)-g(x) &= 2(x+5)^2-(x^2+9x+21) \\ &= 2(x^2+25+10x)-(x^2+9x+21) \\ &= 2x^2+50+20x-x^2-9x-21 \\ &= x^2+11x+29 \end{aligned}$$

The correct choice is **G**.

66. **GRIDDED RESPONSE** In his first three years of coaching basketball at North High School, Coach Lucas' team won 8 games the first year, 17 games the second year, and 6 games the third year. How many games does the team need to win in the fourth year so the coach's average will be 10 wins per year?

SOLUTION:
Let x be the number of games to be won in the fourth year.

So:

$$\frac{8+17+6+x}{4}=10$$
$$31+x=40$$
$$x=9$$

Therefore, the team should win 9 games in the fourth year so that the coach's average will be 10 wins per year.

67. **SAT/**ACT What is the value of $f\ [g(6)]$ if $f(x) = 2x + 4$ and $g(x) = x^2 + 5$?

A 38

B 43

C 57

D 86

E 261

SOLUTION:

$$\begin{aligned} f\left[g(x)\right] &= f\left(x^2+5\right) \\ &= 2\left(x^2+5\right)+4 \\ &= 2x^2+10+4 \\ &= 2x^2+14 \end{aligned}$$

Substitute $x = 6$.

$$\begin{aligned} f\left[g(6)\right] &= 2(6)^2+14 \\ &= 72+14 \\ &= 86 \end{aligned}$$

The correct choice is **D**.

Find all rational zeros of each function.

68. $f(x) = 2x^3 - 13x^2 + 17x + 12$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of 12 and q is a factor of 2.

The possible values of p: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$
The possible values of q: $\pm1, \pm2$
The possible values of $\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \pm\frac{3}{2}$

$$\begin{array}{r|rrrr} 3 & 2 & -13 & 17 & 12 \\ & 0 & 6 & -21 & -12 \\ \hline & 2 & -7 & -4 & \boxed{0} \end{array}$$

The depressed polynomial is $2x^2 - 7x - 4$.
Solve the quadratic equation $2x^2 - 7x - 4$.

$$\begin{aligned} 2x^2 - 7x - 4 &= 0 \\ 2x^2 + x - 8x - 4 &= 0 \\ x(2x+1) - 4(2x+1) &= 0 \\ (2x+1)(x-4) &= 0 \\ 2x = -1 \text{ or } x &= 4 \\ x &= -\frac{1}{2} \end{aligned}$$

The zeros of the function are $-\frac{1}{2}$, 3 and 4.

69. $f(x) = x^3 - 3x^2 - 10x + 24$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of 24 and q is a factor of 1.

The possible values of p: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

The possible values of q: ±1

The possible values of $\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & 24 \\ & 0 & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & \underline{|0} \end{array}$$

Therefore:

$(x-2)(x^2 - x - 12) = 0$

$(x-2)(x-4)(x+3) = 0$

$x = 2, x = 4, x = -3$

The zeros of the function are –3, 2 and 4.

70. $f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of 24 and q is a factor of 1.

The possible values of p: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

The possible values of q: ±1

The possible values of $\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & -7 & 34 & -24 \\ & 0 & 1 & -3 & -10 & 24 \\ \hline & 1 & -3 & -10 & 24 & \underline{|0} \end{array}$$

$(x-1)(x^3 - 3x^2 - 10x + 24) = 0$

The depressed polynomial is $x^3 - 3x^2 - 10x + 24$.

Solve $x^3 - 3x^2 - 10x + 24 = 0$

$x^3 - 3x^2 - 10x + 24 = 0$

$(x-2)(x^2 - x - 12) = 0$

$x = 2, x = 4, x = -3$

Therefore, the roots are $x = 1$, $x = 2$, $x = 4$, and $x = -3$.

71. $f(x)=2x^3-5x^2-28x+15$

SOLUTION:

If $\frac{p}{q}$ is a rational zero, then p is a factor of 24 and q is a factor of 1.

The possible values of p: $\pm1,\pm3,\pm5,\pm15$
The possible values of q: $\pm1,\pm2$
The possible values of $\frac{p}{q}$: $\pm1,\pm3,\pm5,\pm15,\pm\frac{1}{2},\pm\frac{3}{2},\pm\frac{5}{2},\pm\frac{15}{2}$

$$\begin{array}{r|rrrr} 5 & 2 & -5 & -28 & 15 \\ & 0 & 10 & 25 & -15 \\ \hline & 2 & 5 & -3 & \underline{|0} \end{array}$$

The depressed polynomial is $2x^2+5x-3$.

$$(x-5)(2x^2+5x-3)=0$$
$$(x-5)(x+3)(2x-1)=0$$
$$x=5,\ x=-3,\ x=\frac{1}{2}$$

The zeros of the function is $-3,5,\frac{1}{2}$.

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

72. $f(x)=2x^4-x^3+5x^2+3x-9$

SOLUTION:
Use Descartes' Rule.

$$f(x)=2x^4-x^3+5x^2+3x-9$$
$$f(-x)=2x^4+x^3+5x^2-3x-9$$

There are three sign changes in $f(x)$. Therefore, the number of possible positive real zeros is 3 or 1.
There is only one sign change in $f(-x)$. Therefore, the number of possible negative real zero is 1.
Therefore, the number of possible imaginary zero is 2 or 0.

73. $f(x) = -4x^4 - x^2 - x + 1$

SOLUTION:

$$f(x) = -4x^4 - x^2 - x + 1$$
$$f(-x) = -4x^4 - x^2 + x + 1$$

There is only one sign change in $f(x)$. Therefore, the number of possible positive real zero is 1.
There is only one sign change in $f(-x)$. Therefore, the number of possible negative real zero is 1.
Therefore, the number of possible imaginary zeros is 2.

74. $f(x) = 3x^4 - x^3 + 8x^2 + x - 7$

SOLUTION:

$$f(x) = 3x^4 - x^3 + 8x^2 + x - 7$$
$$f(-x) = 3x^4 + x^3 + 8x^2 - x - 7$$

There are three sign changes in $f(x)$. Therefore, the number of possible positive real zeros is 3 or 1.
There is only one sign change in $f(-x)$. Therefore, the number of possible negative real zero is 1.
Therefore, the number of possible imaginary zero is 2 or 0.

75. $f(x) = 2x^4 - 3x^3 - 2x^2 + 3$

SOLUTION:

$$f(x) = 2x^4 - 3x^3 - 2x^2 + 3$$
$$f(-x) = 2x^4 + 3x^3 - 2x^2 + 3$$

There are two sign changes in $f(x)$. Therefore, the number of possible positive real zeros is 2 or 0.
There is only one sign change in $f(-x)$. Therefore, the number of possible negative real zeros is 2 or 0.
Therefore, the number of possible imaginary zeros is 4 or 2 or 0.

76. **MANUFACTURING** A box measures 12 inches by 16 inches by 18 inches. The manufacturer will increase each dimension of the box by the same number of inches and have a new volume of 5985 cubic inches. How much should be added to each dimension?

SOLUTION:

Let *x* inches be the increase in each dimension of the box.
The new volume of the box is given by:

$$V_2 = (12 + x)(16 + x)(18 + x)$$

Substitute 5965 for V_2.

$$(12 + x)(16 + x)(18 + x) = 5985$$
$$x^3 + 46x^2 + 696x - 2529 = 0$$

$$\begin{array}{r|rrrr} 3 & 1 & 46 & 696 & -2529 \\ & 0 & 3 & 147 & 2529 \\ \hline & 1 & 49 & 843 & \lfloor 0 \end{array}$$

Therefore, 3 inches should be added to each dimension.

Solve each system of equations.

77. $$\begin{aligned} x+4y-z&=6\\ 3x+2y+3z&=16\\ 2x-y+z&=3 \end{aligned}$$

SOLUTION:

$$x=\frac{\begin{vmatrix} 6 & 4 & -1\\ 16 & 2 & 3\\ 3 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & -1\\ 3 & 2 & 3\\ 2 & -1 & 1 \end{vmatrix}} =\frac{24}{24} =1$$

$$y=\frac{\begin{vmatrix} 1 & 6 & -1\\ 3 & 16 & 3\\ 2 & 3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & -1\\ 3 & 2 & 3\\ 2 & -1 & 1 \end{vmatrix}} =\frac{48}{24} =2$$

$$z=\frac{\begin{vmatrix} 1 & 4 & 6\\ 3 & 2 & 16\\ 2 & -1 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 4 & -1\\ 3 & 2 & 3\\ 2 & -1 & 1 \end{vmatrix}} =\frac{72}{24} =3$$

The solution is (1, 2, 3).

78. $$\begin{aligned} 2a + b - c &= 5 \\ a - b + 3c &= 9 \\ 3a - 6c &= 6 \end{aligned}$$

SOLUTION:

$$x = \frac{\begin{vmatrix} 5 & 1 & -1 \\ 9 & -1 & 3 \\ 6 & 0 & -6 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 3 \\ 3 & 0 & -6 \end{vmatrix}} = \frac{96}{24} = 4$$

$$y = \frac{\begin{vmatrix} 2 & 5 & -1 \\ 1 & 9 & 3 \\ 3 & 6 & -6 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 3 \\ 3 & 0 & -6 \end{vmatrix}} = \frac{-48}{24} = -2$$

$$z = \frac{\begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 9 \\ 3 & 0 & 6 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 3 \\ 3 & 0 & -6 \end{vmatrix}} = \frac{24}{24} = 1$$

The solution is (4, –2, 1).

79. $$\begin{aligned} y+z&=4\\ 2x+4y-z&=-3\\ 3y&=-3 \end{aligned}$$

SOLUTION:

$$\begin{aligned} 3y&=-3\\ y&=-1 \end{aligned}$$

Substitute $y = -1$ in the equation $y+z=4$.

$$\begin{aligned} -1+z&=4\\ z&=4+1\\ z&=5 \end{aligned}$$

Substitute $y = -1$ and $z = 5$ in the equation $2x+4y-z=-3$.

$$\begin{aligned} 2x+4y-z&=-3\\ 2x+4(-1)-5&=-3\\ 2x-4-5&=-3\\ 2x-9&=-3\\ 2x&=-3+9\\ 2x&=6\\ x&=3 \end{aligned}$$

The solution is (3, –1, 5).

80. **INTERNET** A webmaster estimates that the time, in seconds, to connect to the server when n people are connecting is given by $t(n) = 0.005n + 0.3$. Estimate the time to connect when 50 people are connecting.

SOLUTION:

Replace n with 50.

$$\begin{aligned} t(50)&=0.005(50)+0.3\\ &=0.25+0.3\\ &=0.55 \end{aligned}$$

It takes 0.55 second to connect 50 people.

6-1 Operations on Functions

Solve each equation or formula for the specified variable.

81. $5x - 7y = 12$, for x

SOLUTION:

$$5x = 12 + 7y$$
$$x = \frac{12 + 7y}{5}$$

82. $3x^2 - 6xy + 1 = 4$, for y

SOLUTION:

$$-6xy = 4 - 1 - 3x^2$$
$$y = \frac{3 - 3x^2}{-6x}$$
$$= \frac{1 - x^2}{-2x}$$

83. $4x + 8yz = 15$, for x

SOLUTION:

$$4x = 15 - 8yz$$
$$x = \frac{15 - 8yz}{4}$$

84. $D = mv$, for m

SOLUTION:

$$\frac{D}{v} = \frac{mv}{v}$$
$$\frac{D}{v} = m$$

85. $A = k^2 + b$, for k

SOLUTION:

$$A - b = k^2$$
$$k = \pm\sqrt{A - b}$$

86. $(x + 2)^2 - (y + 5)^2 = 4$, for y

SOLUTION:

$$(y + 5)^2 = (x + 2)^2 - 4$$

Use the Square Root Property.

$$y + 5 = \pm\sqrt{(x + 2)^2 - 4}$$
$$y = \pm\sqrt{(x + 2)^2 - 4} - 5$$